



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1 (a) Show that

$$\tan(r+1) - \tan r = \frac{\sin 1}{\cos(r+1)\cos r}. \quad [2]$$

$$\text{Let } u_r = \frac{1}{\cos(r+1)\cos r}.$$

- (b)** Use the method of differences to find $\sum_{r=1}^n u_r$. [3]

- (c) Explain why the infinite series $u_1 + u_2 + u_3 + \dots$ does not converge. [1]

- 2** The cubic equation $2x^3 - 4x^2 + 3 = 0$ has roots α, β, γ . Let $S_n = \alpha^n + \beta^n + \gamma^n$.

- (a) State the value of S_1 and find the value of S_2 .

[3]

- (b) (i)** Express S_{n+3} in terms of S_{n+2} and S_n .

[1]

- (ii) Hence, or otherwise, find the value of S_4 .

[2]

- (c) Use the substitution $y = S_1 - x$, where S_1 is the numerical value found in part (a), to find and simplify an equation whose roots are $\alpha + \beta$, $\beta + \gamma$, $\gamma + \alpha$. [3]

- (d) Find the value of $\frac{1}{\alpha+\beta} + \frac{1}{\beta+\gamma} + \frac{1}{\gamma+\alpha}$. [2]

- 3 (a) Prove by mathematical induction that, for all positive integers n ,

$$\sum_{r=1}^n (5r^4 + r^2) = \frac{1}{2} n^2 (n+1)^2 (2n+1). \quad [6]$$

- (b)** Use the result given in part **(a)** together with the List of formulae (MF19) to find $\sum_{r=1}^n r^4$ in terms of n , fully factorising your answer. [3]

- 4** The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 2 & k & k \\ 5 & -1 & 3 \\ 1 & 0 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \quad \mathbf{C} = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 0 \end{pmatrix},$$

where k is a real constant.

- (a) Find \mathbf{CAB} .

[3]

- (b) Given that \mathbf{A} is singular, find the value of k .

[3]

- (c) Using the value of k from part (b), find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by **CAB**. [5]

- 5** The curve C has polar equation $r = \frac{1}{\pi - \theta} - \frac{1}{\pi}$, where $0 \leq \theta \leq \frac{1}{2}\pi$.

(a) Sketch C .

[3]

(b) Show that the area of the region bounded by the half-line $\theta = \frac{1}{2}\pi$ and C is $\frac{3 - 4\ln 2}{4\pi}$.

[6]

- 6 The lines l_1 and l_2 have equations $\mathbf{r} = -\mathbf{i} - 2\mathbf{j} + \mathbf{k} + s(2\mathbf{i} - 3\mathbf{j})$ and $\mathbf{r} = 3\mathbf{i} - 2\mathbf{k} + t(3\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ respectively.

The plane Π_1 contains l_1 and the point P with position vector $-2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$.

- (a) Find an equation of Π_1 , giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. [2]

The plane Π_2 contains l_2 and is parallel to l_1 .

- (b) Find an equation of Π_2 , giving your answer in the form $ax + by + cz = d$. [4]

- (c) Find the acute angle between Π_1 and Π_2 . [5]

- (d) The point Q is such that $\overrightarrow{OQ} = -5\overrightarrow{OP}$.

Find the position vector of the foot of the perpendicular from the point Q to Π_2 .

[4]

- 7 The curve C has equation $y = \frac{x^2 - x - 3}{1 + x - x^2}$.

(a) Find the equations of the asymptotes of C .

[2]

(b) Find the coordinates of any stationary points on C .

[3]

- (c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

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- (d) Sketch the curve with equation $y = \left| \frac{x^2 - x - 3}{1 + x - x^2} \right|$ and find in exact form the set of values of x for which $\left| \frac{x^2 - x - 3}{1 + x - x^2} \right| < 3$.

[6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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